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SB-3451

M. A. / M. Sc. (Part I) Examination

March / April – 2011

Mathematics : Paper-403

(Topology)

(Old Course)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. A. / M. Sc. (Part I)**

Name of the Subject :
☛ **Mathematics : Paper-403**

☛ Subject Code No. : **3 4 5 1** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Answer all question.
(3) Figures to the right indicate marks of the question.
(4) Follow usual notation.

1 (a) Let X be a metric space. Prove that a subset F of X is closed if and only if its complement F' is open in X. **5**

(b) Let X be a metric space. Prove that a subset G of X is open if and only if it is a union of open spheres. **5**

(c) Let X be a metric space with metric d. Show that d_1 defined by $d_1(x,y)=\frac{d(x,y)}{1+d(x,y)}$ is also a metric on X. **4**

OR

1 (a) Let X be a metric space, and let Y be a subspace of X. Then prove that Y is complete $\Leftrightarrow$ it is closed. **5**

(b) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous at x_o if and only if $x_n \to x_o \Rightarrow f(X_n) \to f(X_o)$. **5**

(c) If f and g are continuous real function defined on a metric space X, then prove that f+g and αf are also continuous, where α is any real number. 4

2 (a) Define a second countable space and prove that if a non-empty set G in X is represented as the union of a class $\{G_i\}$ of open sets, then G can be represented as a countable union of G_i's; where X is a second countable space. **5**

(b) Let X be a second countable space. Prove that any open base for X has countable subclass which is also an open base. **5**

(c) Let T_1 and T_2 be two topologies on a non-empty set X. What can you say about $T_1 \cup T_2$? Justify your answer. 4

OR

2 (a) Prove that every separable metric space is second countable. **5**

(b) Let X be a topological space and A a subset of X. Prove that (i) Closure of A = $A \cup D(A)$; and **5**

(ii) A is closed $\Leftrightarrow A \supseteq D(A)$.

(c) Let X be a topological space, and let Y and Z be subspaces of X such that $Y \subseteq Z$. Show that the topology which Y has as a subspace of X is the same as that which it has as a subspace of Z. 4

3 (a) Prove that every closed and bounded sbuspace of the real line compact. **5**

(b) State finite intersection property and prove that a topological space is compact if and only if every class of closed sets with the finite intersection property has non-empty intersection. **5**

(c) Prove that every compact subspace of the real line is closed and bounded. **4**

OR

3 (a) Prove that in a sequentially compact metric space, every open cover has a Lebesgue number. 6

(b) Prove that every sequentially compact metric space is compact. **4**

(c) Prove that a second countable space is countably compact $\Leftrightarrow$ it is compact. **4**

4 (a) Prove that in a Hausdorff space, any point and disjoint compact subspace can be separated by open sets, in the sense that they have disjoint neighborhoods. **5**

(b) Define normal space and prove that every compact Hausdorff space is normal. **5**

(c) If X is a Hausdorff space, show that every convergent sequence in X has a unique limit. **4**

OR

4 (a) Let X be a normal space, and let A and B be disjoint closed subspaces of X. Then prove that there exists a continuous real function f defined on X, all of whose values lie in the closed unit interval [0,1], such that f(A)=0 and f(B)=1. **7**

(b) Prove that the compact subspace of T_2 space is closed. **4**

(c) Prove that any subspace of a T_2 space is also a T_2 space. **3**

5 (a) Prove that a subspace of the real line R is connected if and only if it is an interval. **6**

(b) Prove that the space R^n is connected. **4**

(c) Prove that the components of a totally disconnected space are its points. 4

OR

5 (a) Let X be a topological space. If $\{A_i\}$ is a non-empty class of connected subspaces of X such that $\cap_i A_i$ is non-empty then prove that $A = \cup_i A_i$ is also a connected subspace of X. **5**

(b) Let X be a topological space. Then prove that **5**

(i) each connected subspace of X is contained in a component of X.

(ii) a connected subspace of X which is both open and closed is a component of X.

(c) Let X be a topological space. Then prove that every closed sub-algebra of C(X, R) is also a closed sublattice of C(X, R). 4